# Managing Cycle Inventories

Matching Supply and Demand

# Outline

- Why to hold cycle inventories?
- Economies of scale to reduce fixed costs per unit.
- Joint fixed costs for multiple products
- Long term quantity discounts
- Short term quantity discounts: Promotions

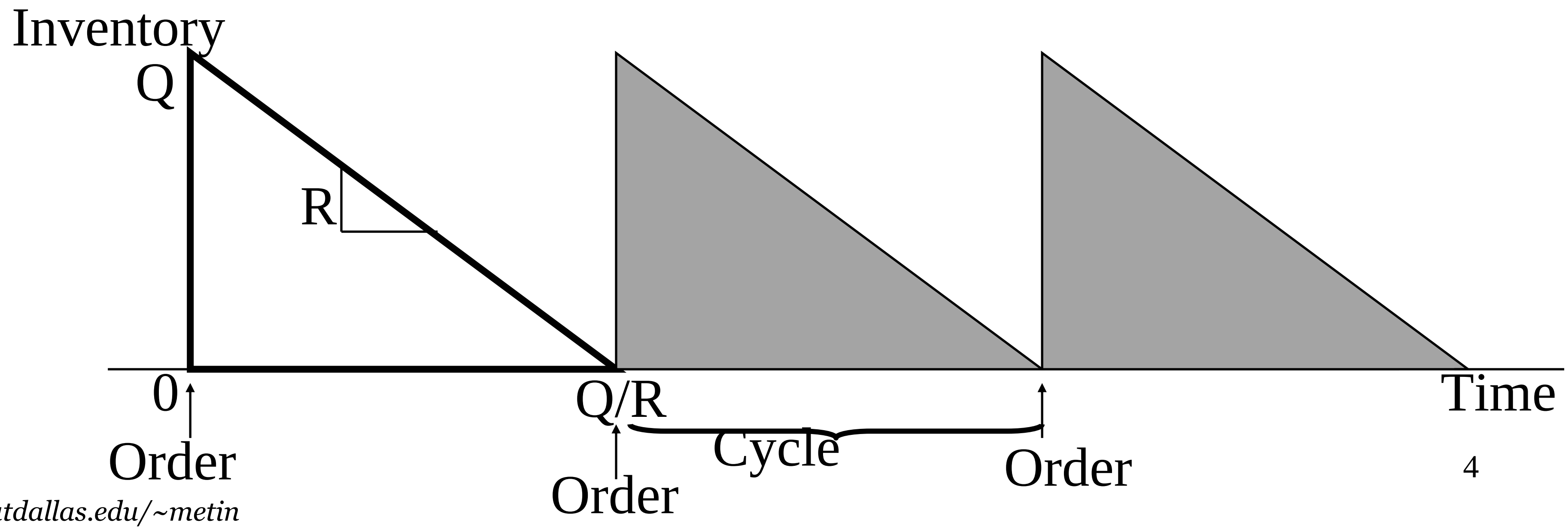
# Role of Inventory in the Supply Chain

- **Overstocking: Amount available exceeds demand**
  - **Liquidation, Obsolescence, Holding**
- **Understocking: Demand exceeds amount available**
  - **Lost margin and future sales**

**Goal: Matching supply and demand**

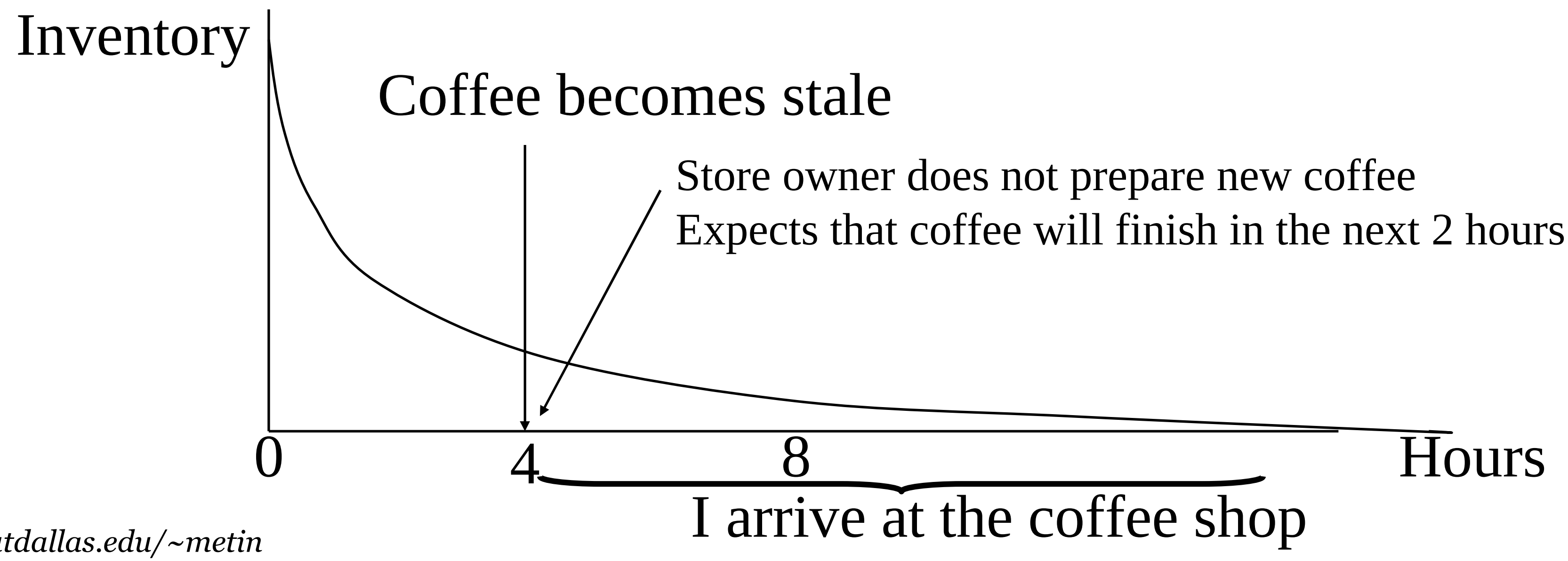
# Batch or Lot size

- **Batch = Lot = quantity of products bought / produced together**
  - **But not simultaneously, since production can not be simultaneous**
  - **Q: Lot size. R: Demand per time.**
- **Consider sales at a Jean's retailer with demand of 10 jeans per day and an order size of 100 jeans.**
  - **Q=100. R=10/day.**



# Demand affected by visibility

- Demand is higher when the inventory is higher and is smaller when the inventory is smaller.
  - When I am buying coffee, it is often not fresh. Why?
  - Fresh coffee is consumed fast but stale coffee is not.
  - Or because:



# Batch or Lot size

- Cycle inventory=Average inventory held during the cycle
  =Q/2=50 jean pairs
- Average flow time
  – Remember Little's law
  =(Average inventory)/(Average flow rate)=(Q/2)/R=5 days
- Long flow times make a company vulnerable to product / technology changes
- Lower cycle inventory decreases working (operating) capital needs and space requirements for inventory
- Then, why not to set Q as low as possible?

# Why to order in lots?

- Fixed ordering cost: S
  - Increase the lot size to decrease the fixed ordering cost per unit
- Material cost per unit: C
- Holding cost: Cost of carrying 1 unit in the inventory: H
  - H=h.C
  - h: carrying $1 in the inventory > interest rate
- Lot size is chosen by trading off holding costs against fixed ordering costs (and sometimes material costs).
- Where to shop from:

| | Fixed cost (driving) | Material cost |
|---|---|---|
| Convenience store | low | HIGH |
| Sam’s club | HIGH | low |

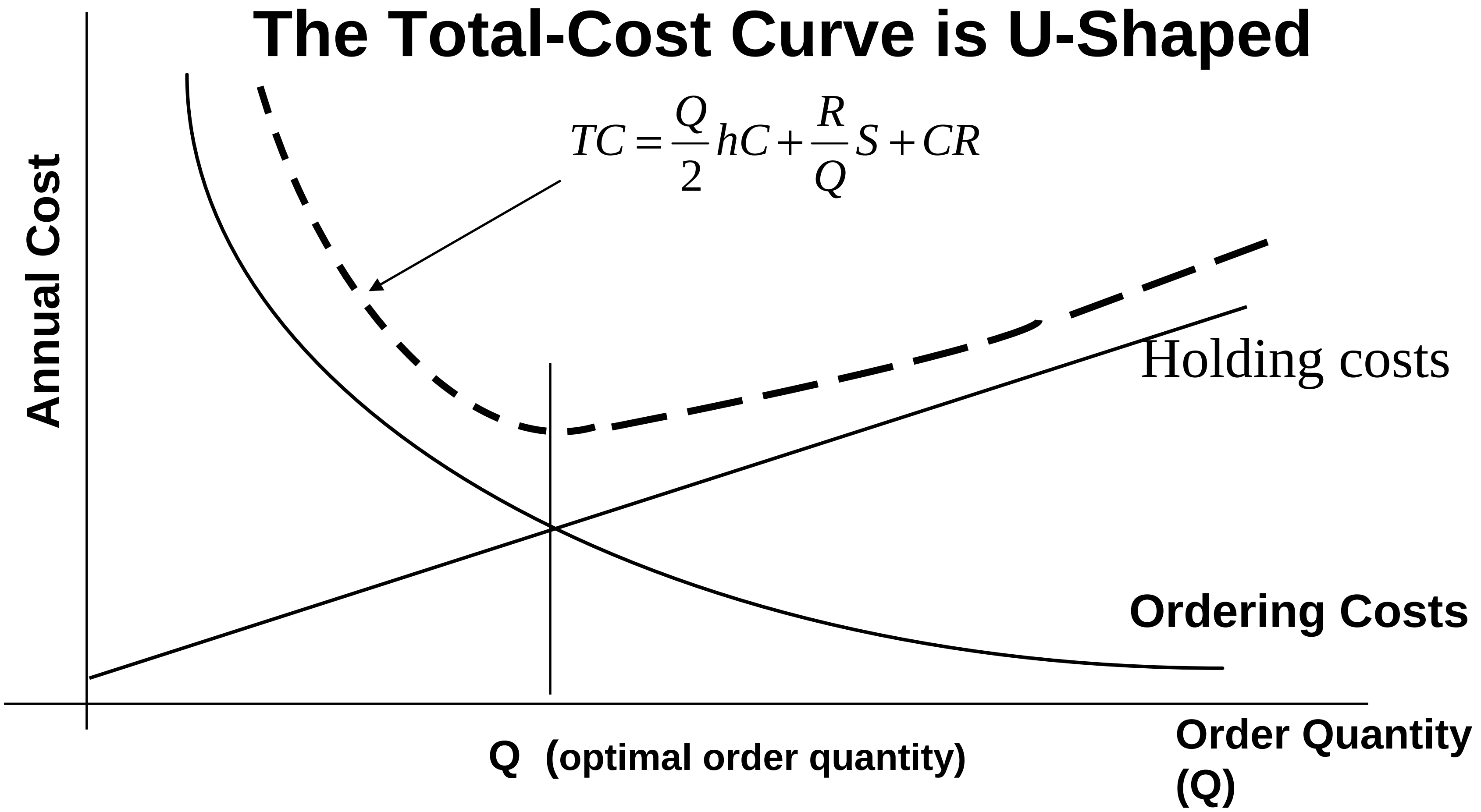
# Economic Order Quantity - EOQ

**TC = Annual carrying cost + Annual ordering cost + Purchasing cost**

$$TC = \frac{Q}{2}hC + \frac{R}{Q}S + CR$$

Total cost is simple function of the lot size Q.
Note that we can drop the last term, it is not affected by the choice of Q.

# Cost Minimization Goal



# Deriving the EOQ

Using calculus, we take the derivative of the total cost function and set the derivative equal to zero and solve for Q. Total cost curve is convex i.e. curvature is upward so we obtain the minimizer.

$$EOQ = \sqrt{\frac{2RS}{hC}} \qquad T = \frac{EOQ}{R} = \sqrt{\frac{2S}{RhC}} \qquad n = \frac{R}{EOQ} = \sqrt{\frac{RhC}{2S}}$$

T: Reorder interval length = EOQ/R.

n: Ordering frequency: number of orders per unit time = R/EOQ.

The total cost curve reaches its minimum where the inventory carrying and ordering costs are equal.

$$\text{Total cost}(Q = EOQ) = \sqrt{2RShC}$$

# EOQ example

Demand, R = 12,000 computers per year. Unit cost, C = $500

Holding cost, h = 0.2.  Fixed cost, S = $4,000/order.

Find EOQ, Cycle Inventory, Average Flow Time, Optimal Reorder Interval and Optimal Ordering Frequency.

Q = 979.79, say 980 computers

Cycle inventory =  Q/2 = 490 units

Average Flow Time = Q/(2R) = 0.49 month

Optimal Reorder interval, T = 0.0816 year = 0.98 month

Optimal ordering frequency, n=12.24 orders per year.

# Key Points from Batching

- In deciding the optimal lot size the trade off is between setup (order) cost and holding cost.

- If demand increases by a factor of 4, it is optimal to increase batch size by a factor of 2 and produce (order) twice as often. Cycle inventory (in units) doubles. *Cycle inventory (in days of demand) halves*.

- If lot size is to be reduced, one has to reduce fixed order cost. To reduce lot size by a factor of 2, order cost has to be reduced by a factor of 4. This is what JIT strives to do.

# Strategies for reducing fixed costs

- In production
  - Standardization / dedicated
  - Simplification
  - Set up out of the production line
- In delivery
  - Third party logistics
  - Aggregating multiple products in a single order
    - Temporal, geographic aggregation
  - Various truck sizes, difficult to manage

# Example: Lot Sizing with Multiple Products

- Demand per year
  - $R_L$ = 12,000; $R_M$ = 1,200; $R_H$ = 120
- Common transportation cost per delivery,
  - $S$ = \$4,000
- Product specific order cost per product in each delivery
  - $s_L$ = \$1,000; $s_M$ = \$1,000; $s_H$ = \$1,000
- Holding cost,
  - $h$ = 0.2
- Unit cost
  - $C_L$ = \$1,000; $C_M$ = \$1,000; $C_H$ = \$1,000

# Delivery Options

- No Aggregation:
  - Each product ordered separately
- Complete Aggregation:
  - All products delivered on each truck
- Tailored Aggregation:
  - Selected subsets of products on each truck

# No Aggregation: Order each product independently

| | *Litepro* | *Medpro* | *Heavypro* |
|---|---|---|---|
| **Demand per year** | **12,000** | **1,200** | **120** |
| **Fixed cost / order** | **$5,000** | **$5,000** | **$5,000** |
| **Optimal order size** | **1,095** | **346** | **110** |
| **Order frequency** | **11.0 / year** | **3.5 / year** | **1.1 / year** |
| **Annual cost** | **$109,544** | **$34,642** | **$0,954** |

**Total cost = $155,140**

# Complete Aggregation: Order all products jointly

- Total ordering cost $S^* = S + s_L + s_M + s_H$ = \$7,000
- n: common ordering frequency
- Annual ordering cost = n S*
- Total holding cost:

$$\frac{R_L}{2n} h C_L + \frac{R_M}{2n} h C_M + \frac{R_H}{2n} h C_H$$

- Total cost:

$$TC(n) = S^* n + \frac{h}{2n}\left(R_L C_L + R_M C_M + R_H C_H\right)$$

$$n^* = \sqrt{\frac{h\left(R_L C_L + R_M C_M + R_H C_H\right)}{2S^*}}$$

# Complete Aggregation: Order all products jointly

| | *Litepro* | *Medpro* | *Heavypro* |
|---|---|---|---|
| **Demand per year** | **12,000** | **1,200** | **120** |
| **Order frequency** | **9.75/year** | **9.75/year** | **9.75/year** |
| **Optimal order size** | **1,230** | **123** | **12.3** |
| **Annual holding cost** | **$61,512** | **$6,151** | **$615** |

**Annual order cost = 9.75×$7,000 = $68,250**
**Annual total cost = $136,528**

Ordering high and low volume items at the same frequency cannot be a good idea.

# Tailored Aggregation: Ordering Selected Subsets

- Example Orders may look like (L,M); (L,H); (L,M); (L,H).
- Most frequently ordered product: L
- M and H are ordered in every other delivery.
- We can associate fixed order cost S with product L because it is ordered every time there is an order.
- Products other than L are associated only with their incremental order costs (s values).

An Algorithm:

Step 1: Identify most frequently ordered product

Step 2: Identify frequency of other products as a relative multiple

Step 3: Recalculate ordering frequency of most frequently ordered product

Step 4: Identify ordering frequency of all products

# Tailored Aggregation: Ordering Selected Subsets

- i is the generic index for products, i is L, M or H.
- Step 1: Find most frequently ordered item:

$$\bar{n}_i = \sqrt{\frac{hC_iR_i}{2(S+s_i)}} \qquad n = \max\{\bar{n}_i\}$$

The frequency of the most frequently ordered item will be modified later. This is an approximate computation.

- Step 2: Relative order frequency of other items, $m_i$

$$\bar{\bar{n}}_i = \sqrt{\frac{hC_iR_i}{2s_i}} \qquad m_i = \left\lceil \frac{n}{\bar{\bar{n}}_i} \right\rceil$$

$m_i$ are relative order frequencies, they must be integers.

# Tailored Aggregation: Ordering Selected Subsets

- Step 3: Recompute the frequency of the most frequently ordered item. This item is ordered in every order whereas others are ordered in every $m_i$ orders. The average fixed ordering cost is: $$S + \sum_i \frac{s_i}{m_i}$$

$$\text{Annual ordering cost} = n(S + \sum_i \frac{s_i}{m_i})$$

$$\text{Annual holding cost} = \sum_i \frac{R_i}{2n / m_i} hC_i$$

$$n^* = \sqrt{\frac{\sum_i R_i m_i h C_i}{2\left(S + \sum_i \frac{s_i}{m_i}\right)}}$$ different than (10.8) on p.263 of Chopra

# Tailored Aggregation: Ordering Selected Subsets

- Step 4: Recompute the ordering frequency $n_i$ of other products:

$$n_i = \frac{n}{m_i}$$

- Total Annual ordering cost: $nS+n_H s_H+n_M s_M+n_L s_L$

- Total Holding cost:

$$\frac{R_L}{2n_L}hC_L + \frac{R_M}{2n_M}hC_M + \frac{R_H}{2n_H}hC_H$$

# Tailored Aggregation: Ordering Selected Subsets

- Step 1:

$$\bar{n}_L = \sqrt{\frac{hC_L R_L}{2(S+s_L)}} = 11,\ \bar{n}_M = 3.5,\ \bar{n}_H = 1.1 \qquad n = \max\{\bar{n}_i\} = 11$$

- Step 2:

$$\bar{\bar{n}}_M = \sqrt{\frac{hC_M R_M}{2s_M}} = 7.7\ ,\ \bar{\bar{n}}_H = 2.4; \quad m_M = \left\lceil \frac{n}{\bar{\bar{n}}_M} \right\rceil = 2\ ,\ m_H = 5$$

Item L is ordered most frequently.
Every other L order contains one M order.
Every 5 L orders contain one H order.
At this step we only now relative frequencies, not the actual frequencies.

# Tailored Aggregation: Ordering Selected Subsets

- Step 3: $n^* = \sqrt{\dfrac{\sum_i hC_i R_i m_i}{2(S + \sum_i \frac{s_i}{m_i})}} = 11.47$
- Step 4:

$$n_M = \frac{n^*}{m_M} = 5.73 \qquad n_H = \frac{n^*}{m_H} = 2.29$$

- Total ordering cost:
  - $nS+n_H s_H+n_M s_M+n_L s_L$=11.47(4000)+11.47(1000)+5.73(1000)+2.29(1000)
- Total holding cost

$$\frac{R_L}{2n_L}hC_L + \frac{R_M}{2n_M}hC_M + \frac{R_H}{2n_H}hC_H$$

$$= \frac{12000}{2(11.47)}(0.2)500 + \frac{1200}{2(5.73)}(0.2)500 + \frac{120}{2(2.29)}(0.2)500$$

# Tailored Aggregation: Order selected subsets

| | *Litepro* | *Medpro* | *Heavypro* |
|---|---|---|---|
| **Demand per year** | **12,000** | **1,200** | **120** |
| **Order frequency** | **11.47/year** | **5.73/year** | **2.29/year** |
| **Optimal order size** | **1046.2** | **104.7** | **26.3** |
| **Annual holding cost** | **$52,810** | **$10,470** | **$2,630** |

**Annual order cost = $65,370**
**Total annual cost = $130,650**

# Lessons From Aggregation

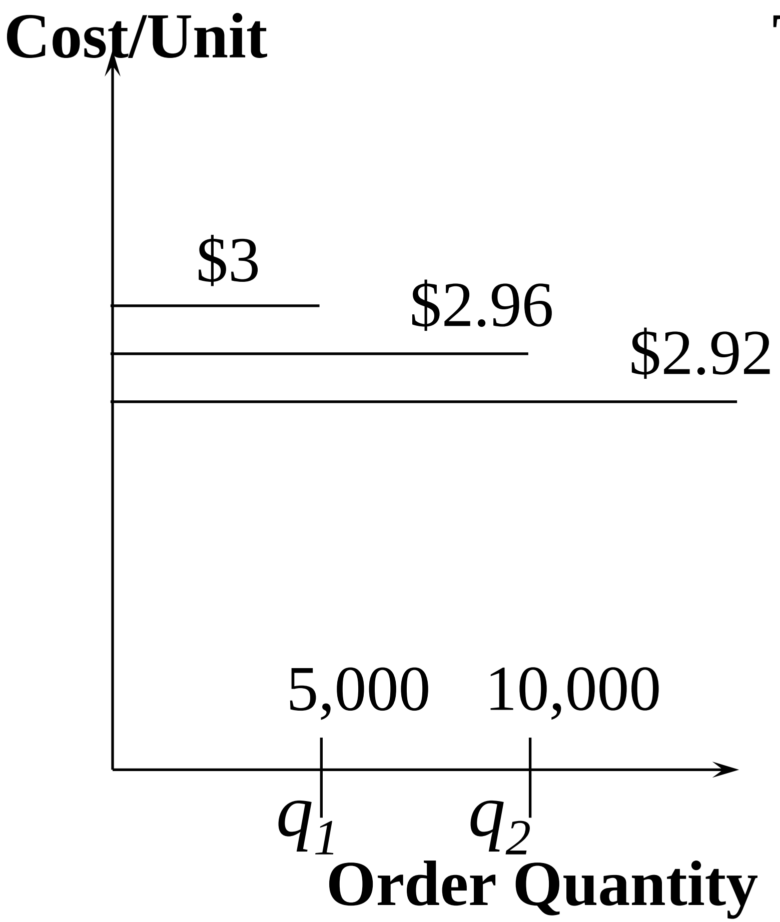
- Information technology can decrease product specific ordering costs.
- Aggregation allows firm to lower lot size without increasing cost
  - Order frequencies without aggregation and with tailored aggregation
    - (11; 3.5; 1.1) vs. (11.47; 5.73; 2.29)
- Complete aggregation is effective if product specific fixed cost is a small fraction of joint fixed cost
- Tailored aggregation is effective if product specific fixed cost is large fraction of joint fixed cost

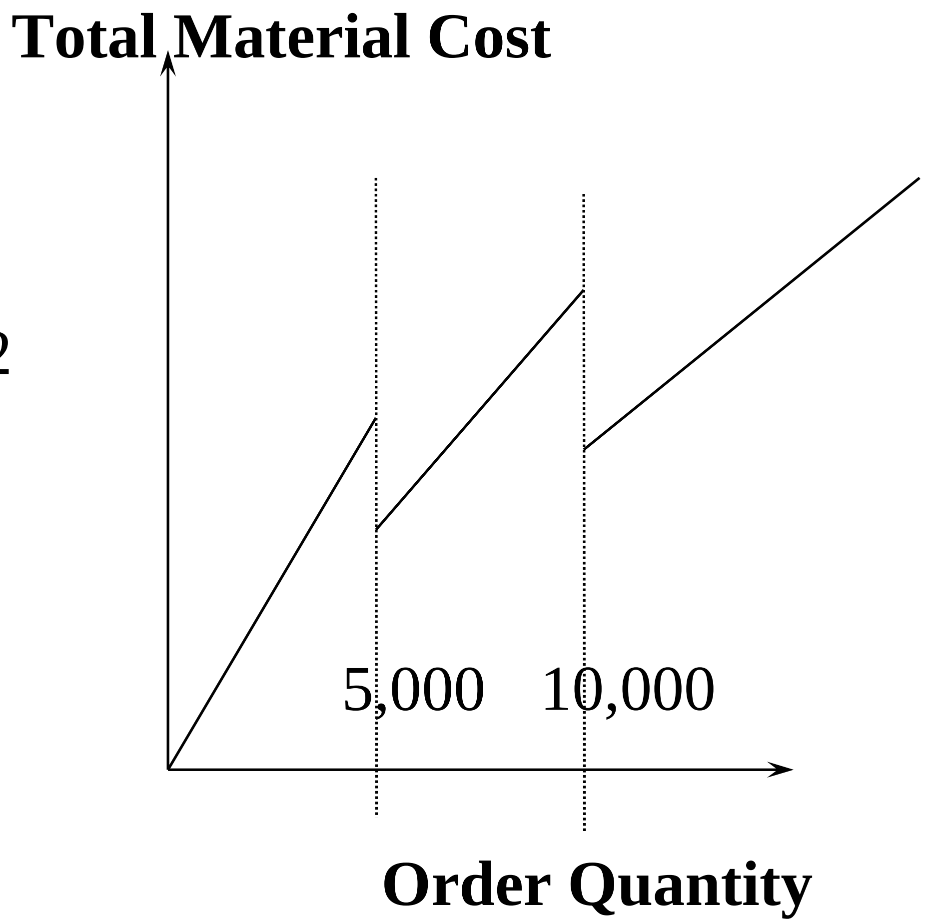
# Quantity Discounts

- Lot size based
  - All units
  - Marginal unit
- Volume based

- *How should buyer react?*
- *What are appropriate discounting schemes?*

# All-Unit Quantity Discounts



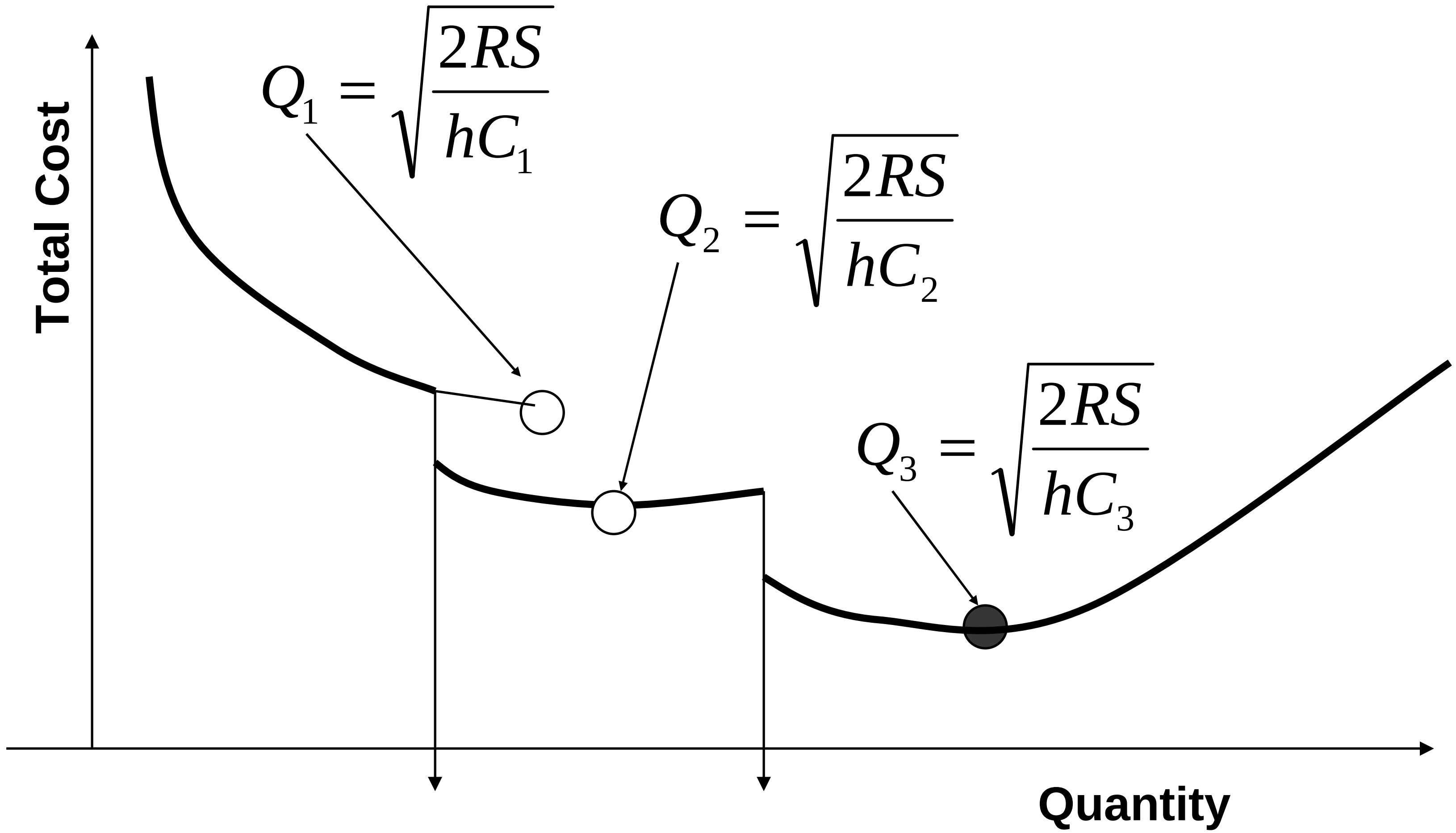



# All-Unit Quantity Discounts

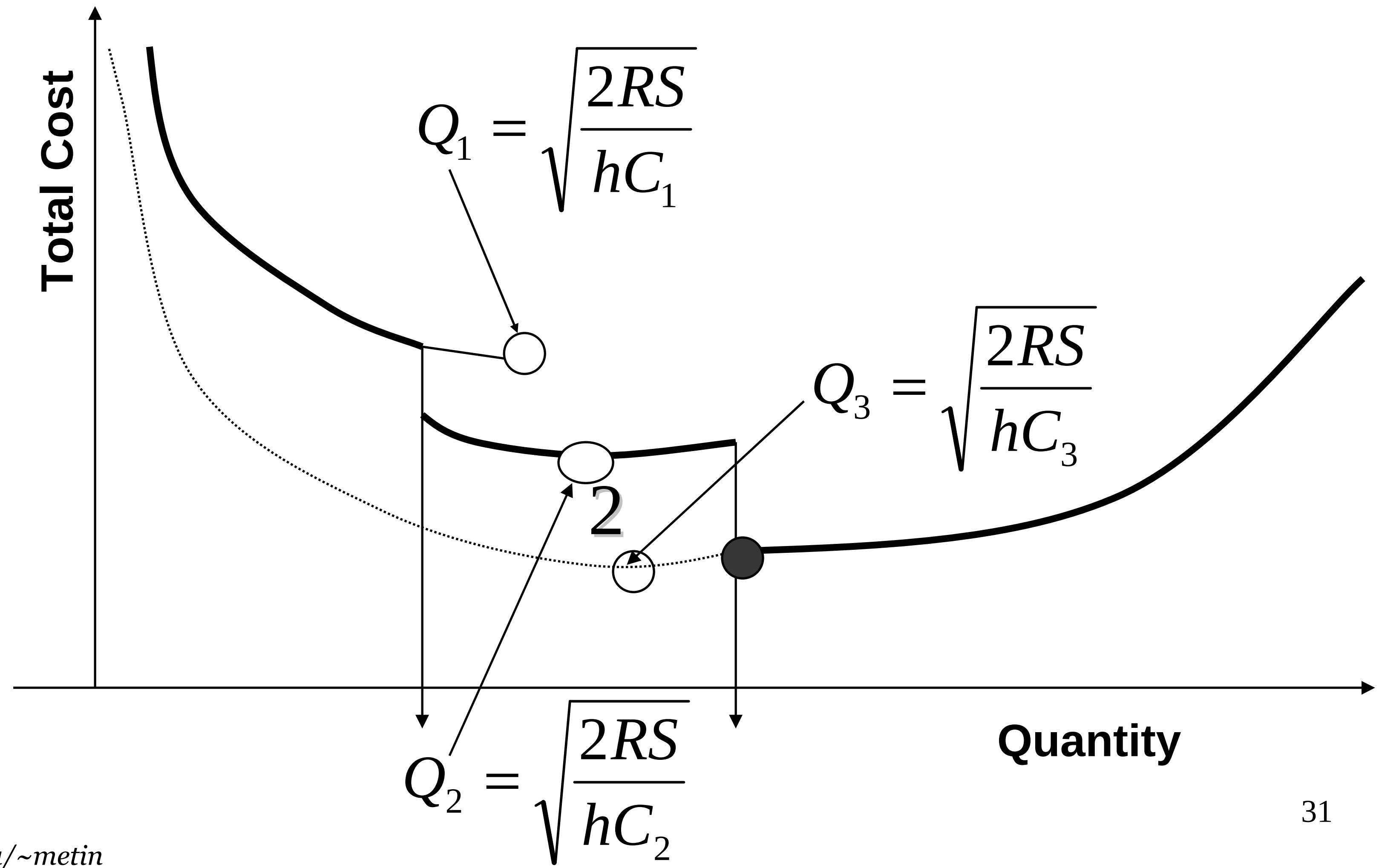
- Find EOQ for price in range $q_i$ to $q_{i+1}$
  - If $q_i \leq \text{EOQ} < q_{i+1}$ ,
    - » Candidate in this range is EOQ, evaluate cost of ordering EOQ
  - If EOQ < $q_i$,
    - » Candidate in this range is $q_i$, evaluate cost of ordering $q_i$
  - If EOQ ≥ $q_{i+1}$ ,
    - » Candidate in this range is $q_{i+1}$, evaluate cost of ordering $q_{i+1}$

- Find minimum cost over all candidates

# Finding Q with all units discount



# Finding Q with all units discount



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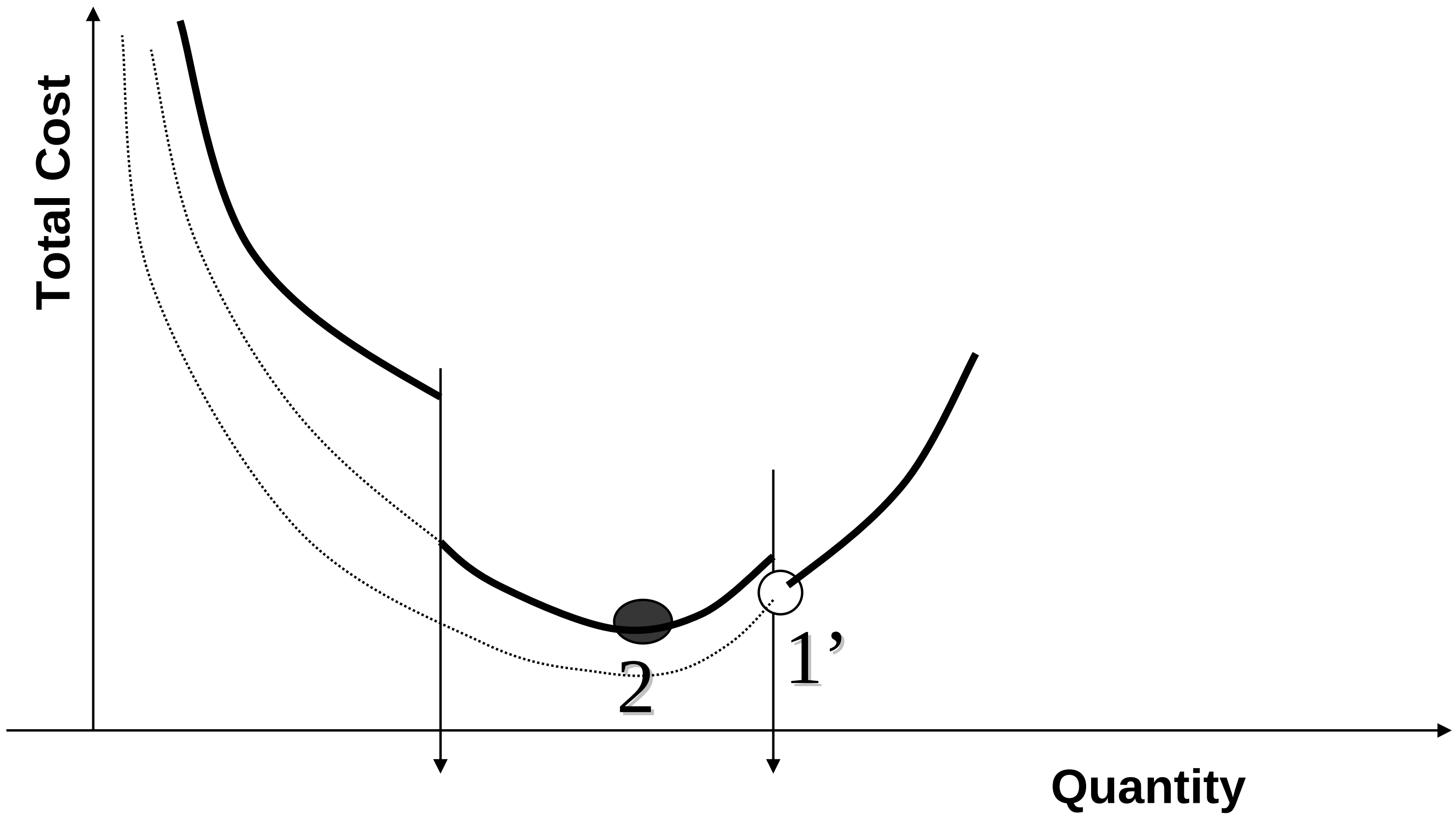
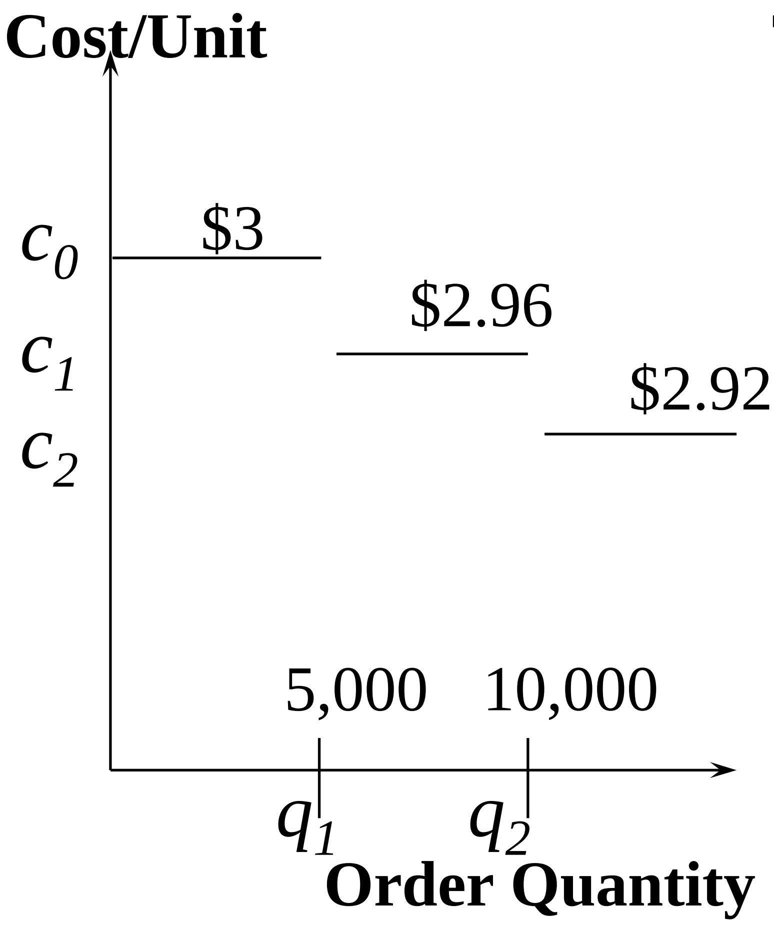
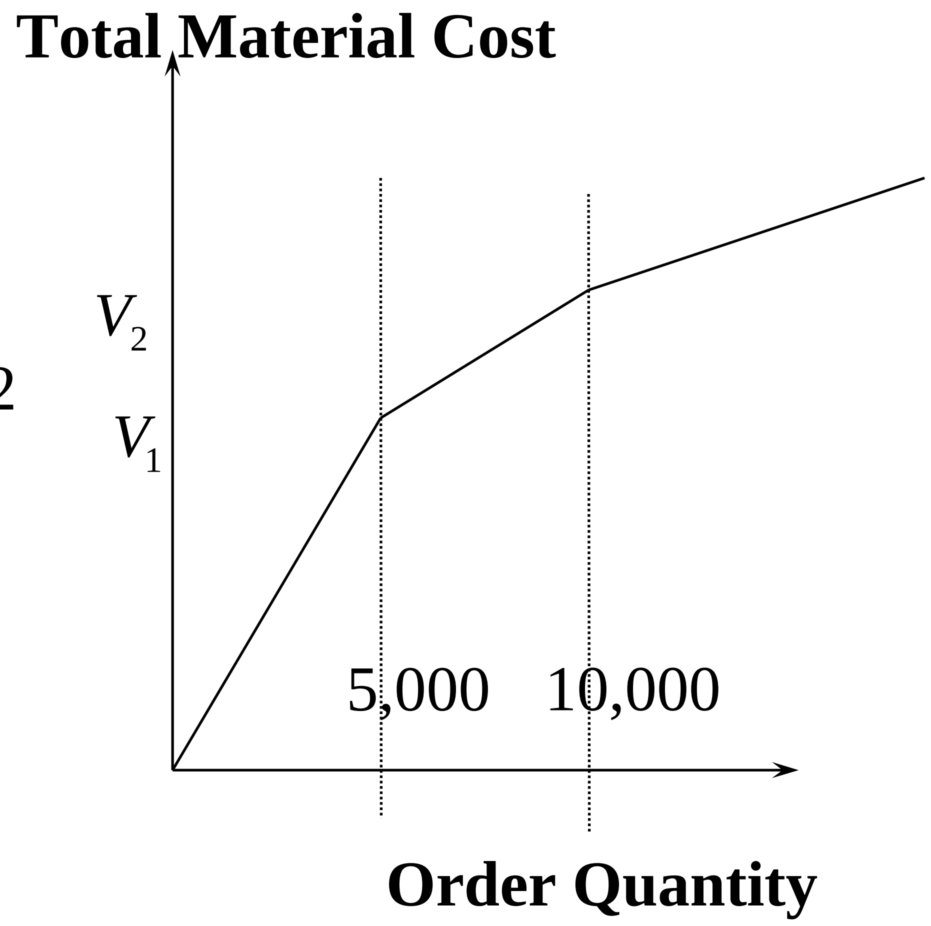


# Marginal Unit Quantity Discounts



# Marginal Unit Quantity Discounts

$V_i$ = Cost of buying exactly $q_i$. $V_0 = 0$.

$$V_i = c_0(q_1 - q_0) + c_1(q_2 - q_1) + \ldots + c_{i-1}(q_i - q_{i-1})$$

If $q_i \le Q \le q_{i+1}$,

$$\text{Annual order cost} = \frac{R}{Q}S$$

$$\text{Annual holding cost} = \left(V_i + (Q - q_i)c_i\right)\frac{h}{2}$$

$$\text{Annual material cost} = \frac{R}{Q}\left(V_i + (Q - q_i)c_i\right)$$

$$\frac{\partial\, \text{Total cost}(Q)}{\partial Q} = -\frac{R}{Q^2}S + c_i\frac{h}{2} - \frac{R}{Q^2}\left(V_i - q_i c_i\right) = 0$$

$$\text{For range } i, \quad EOQ = \sqrt{\frac{2R\left(S + V_i - q_i c_i\right)}{hc_i}}$$

# Marginal Unit Quantity Discounts

$V_i$ = Cost of buying exactly $q_i$. $V_0 = 0$.

$V_i = c_0(q_1 - q_0) + c_1(q_2 - q_1) + \ldots + c_{i-1}(q_i - q_{i-1})$

If $q_i \le Q \le q_{i+1}$,

Annual order cost $= \dfrac{R}{Q} S$

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Annual material cost $= \dfrac{R}{Q}\left(V_i + (Q - q_i)c_i\right)$

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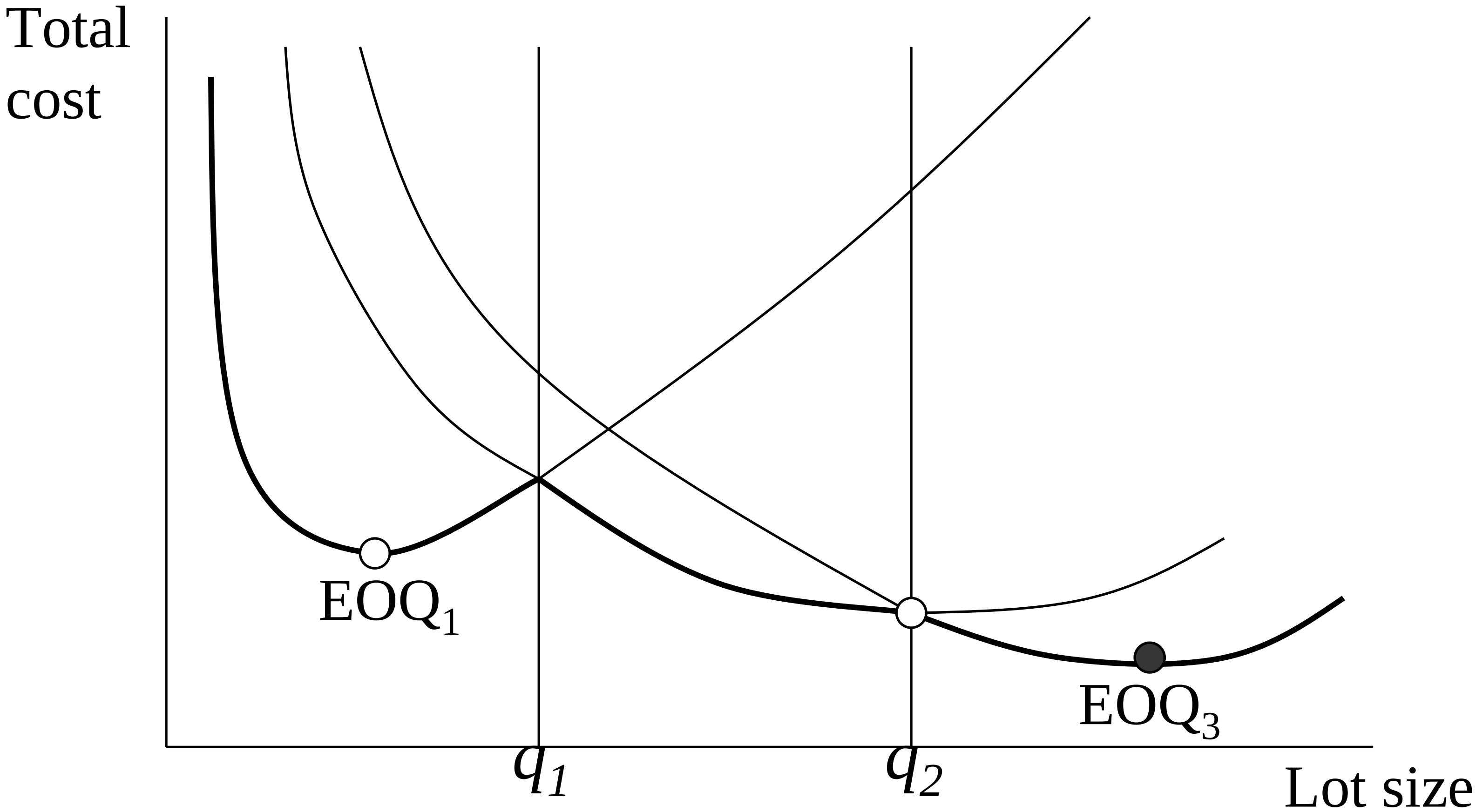
For range $i$, $EOQ = \sqrt{\dfrac{2R\left(S + V_i - q_i c_i\right)}{h c_i}}$

# Marginal-Unit Quantity Discounts

- Find EOQ for price in range $q_i$ to $q_{i+1}$
  - If $q_i \leq$ EOQ $< q_{i+1}$ ,
    - » Candidate in this range is EOQ, evaluate cost of ordering EOQ
  - If EOQ $< q_i$,
    - » Candidate in this range is $q_i$, evaluate cost of ordering $q_i$
  - If EOQ $\geq q_{i+1}$ ,
    - » Candidate in this range is $q_{i+1}$, evaluate cost of ordering $q_{i+1}$

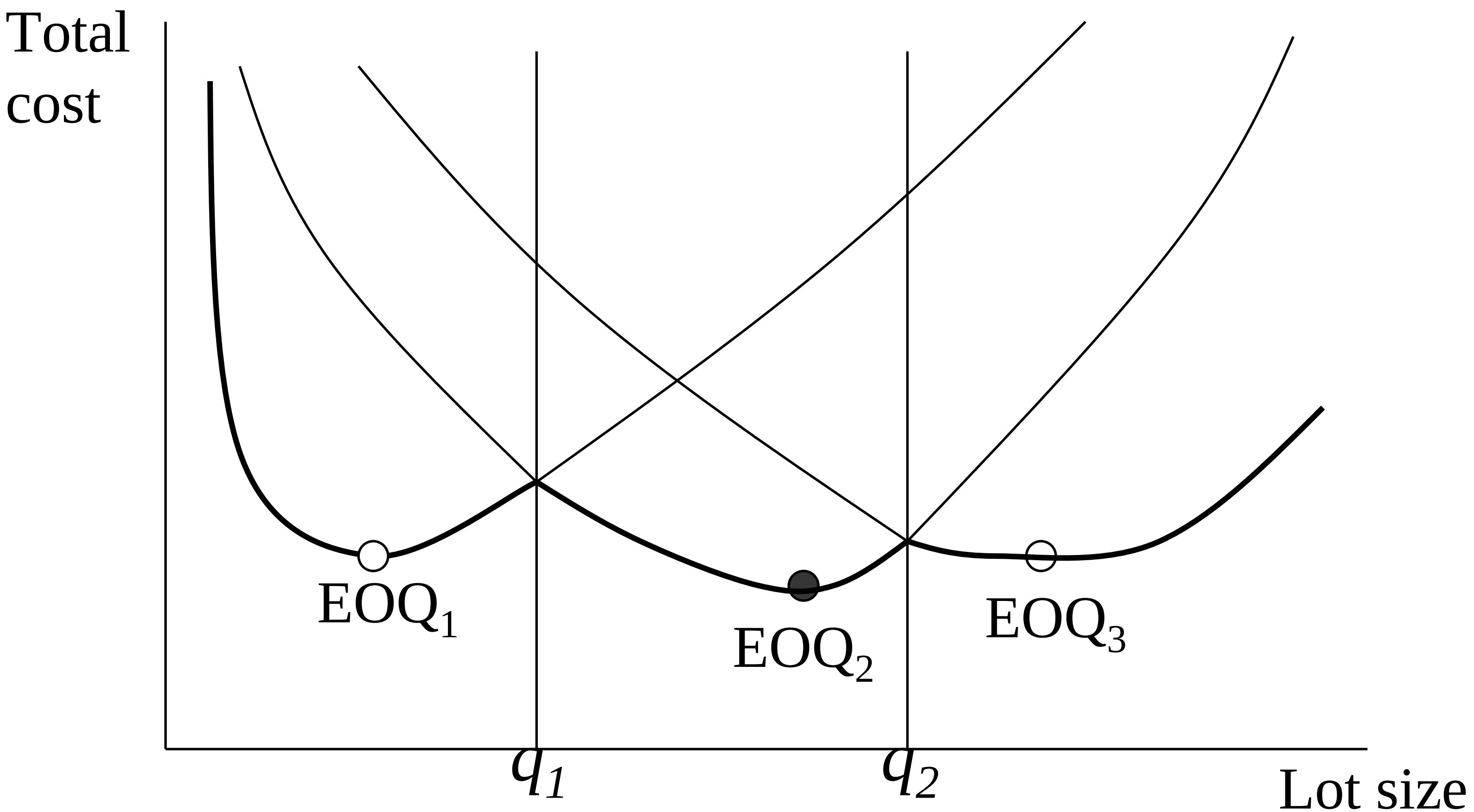
- Find minimum cost over all candidates

# Marginal Unit Quantity Discounts




Compare this total cost graph with that of all unit quantity discounts. Here the cost graph is continuous whereas that of all unit quantity discounts has breaks.

# Marginal Unit Quantity Discounts



# Why Quantity Discounts?

- The lot size that minimizes retailers cost does not necessarily minimize supplier and retailer's cost together.
- Coordination in the supply chain
  - Will supplier and retailer be willing to operate with the same order sizes, frequencies, prices, etc. ? How to ensure this willingness? Via contracts.
  - Quantity discounts given by a supplier to a retailer can motivate the retailer to order as the supplier wishes.

# Coordination for Commodity Products: Supplier and Retailer Coordination

- Consider a supplier S and retailer R pair
- $R$ = 120,000 bottles/year
- $S_R$ = \$100, $h_R$ = 0.2, $C_R$ = \$3
- $S_S$ = \$250, $h_S$ = 0.2, $C_S$ = \$2

Retailer's optimal lot size = 6,324 bottles

Retailer's annual ordering and holding cost = $3,795;

If Supplier uses the retailer's lot size,

  Supplier's annual ordering and holding cost = $6,009

Total annual supply chain cost = $9,804

# Coordination for Commodity Products

- What can the supplier do to decrease supply chain costs? Combine the supplier and the retailer
  - Coordinated lot size: 9,165= $\sqrt{\dfrac{2R(S_S + S_R)}{h(C_S + C_R)}}$
  - Retailer cost = \$4,059; Supplier cost = \$5,106;
  - Supply chain cost = \$9,165. \$639 less than without coordination.

$$\text{Choose } Q_R \text{ by } \text{Minimize RetailerCost}(Q) \text{, then}$$

$$\text{RetailerCost}(Q_R) + \text{SupplierCost}(Q_R) \geq \underset{Q}{\text{Minimize}}\ \text{RetailerSupplierCost}(Q)$$

$$\text{Coordination Savings} = \{\text{RetailerCost}(Q_R) + \text{SupplierCost}(Q_R)\} - \underset{Q}{\text{Minimize}}\ \text{RetailerSupplierCost}(Q)$$

# Coordination via Pricing by the Supplier

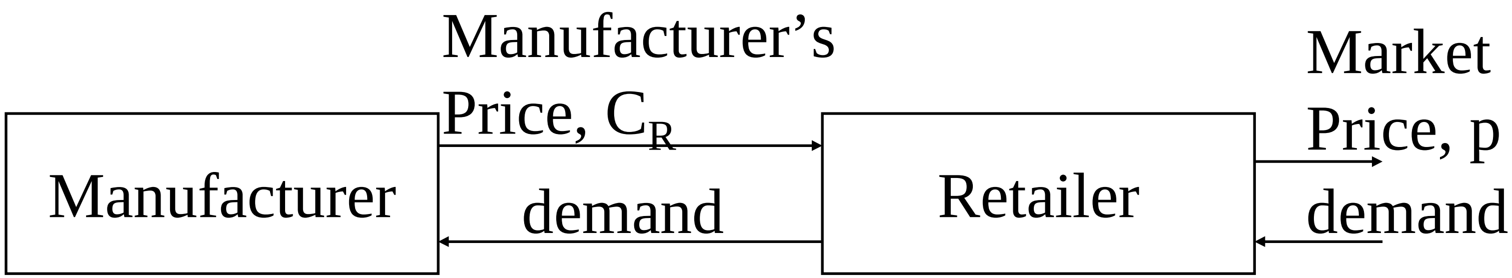
- Effective pricing schemes
  - All unit quantity discount
    - $3 for lots below 9,165
    - $2.9978 for lots of 9,165 or more
  - What is supplier’s and retailer’s cost with the all unit quantity discount scheme? Not the same as before. Who gets the savings due to coordination?
  - Pass some fixed cost to retailer (enough that the retailer raises order size from 6,324 to 9,165)

# Quantity Discounts for a Firm with Market Power (Price dependent demand)

- No inventory related costs
- Demand curve

  $360{,}000 - 60{,}000p$

  Retailer discounts to manipulate the demand
- Retailer chooses the market price p, manufacturer chooses the sales price $C_R$ to the retailer.
- Manufacturing cost $C_M$=\$2/unit



# Quantity Discounts for a Firm with Market Power

- Retailer profit=$(p-C_R)(360{,}000-60{,}000p)$
- Manufacturer profit=$(C_R-C_M)$ $(360{,}000-60{,}000p)$
  - Note $C_M$=\$2
- If each optimizes its own profit:
- Manufacturer assumes that p= $C_R$
  - Sets $C_R$=\$4 to maximize $(C_R-2)$ $(360{,}000-60{,}000C_R)$
- Retailer takes $C_R$=\$4
  - Sets p=\$5 to maximize $(p-4)(360{,}000-60{,}000p)$
- Q=60,000. Manufacturer and retailer profits are \$120K and \$60K respectively. Total SC profit is \$180K.
- Observe that if p=\$4, total SC profits are (4-2)120K=\$240K.
- How to capture 240-180=\$60K?

# Two Part Tariffs and Volume Discounts

- Design a two-part tariff that achieves the coordinated solution.
- Design a volume discount scheme that achieves the coordinated solution.

- Impact of inventory costs
  – Pass on some fixed costs with above pricing

# Two part tariff to capture all the profits

- Manufacturer sells each unit at \$2 but adds a fixed charge of \$180K.
- Retailer profit=(p-2)(360,000-60,000p)-180,00
  - Retailer sets p=\$4 and obtains a profit of \$60K
  - Q=120,000
- Manufacturer makes money only from the fixed charge which is \$180K.
- Total profit is \$240K. Manufacturer makes \$60K more. Retailer’s profit does not change.
- Does the retailer complain?
- Split of profits depend on bargaining power
  - Signaling strength
  - Other alternative buyers and sellers
  - Previous history of negotiations; credibility (of threats)
  - Mechanism for conflict resolution: iterative or at once

# All units discount to capture all profits

- Supplier applies all unit quantity discount:
  - If $0<Q<120{,}000$, $C_R=\$4$
  - Else $C_R=\$3.5$
- If $Q<120{,}000$, we already worked out that $p=\$5$ and $Q=60{,}000$. And the total profit is \$180,000.
- If $Q>=120{,}000$, the retailer chooses $p=\$4.75$ which yields $Q=75{,}000$ and is outside the range. Then $Q=120{,}000$ and $p=\$4$.
- Retailer profit=(4-3.5)120,000=60,000
- Manufacturer profit=(3.5-2)120,000=180,000
- Total SC profits are again \$240K.
- Manufacturer discounts to manipulate the market demand via retailer's pricing.

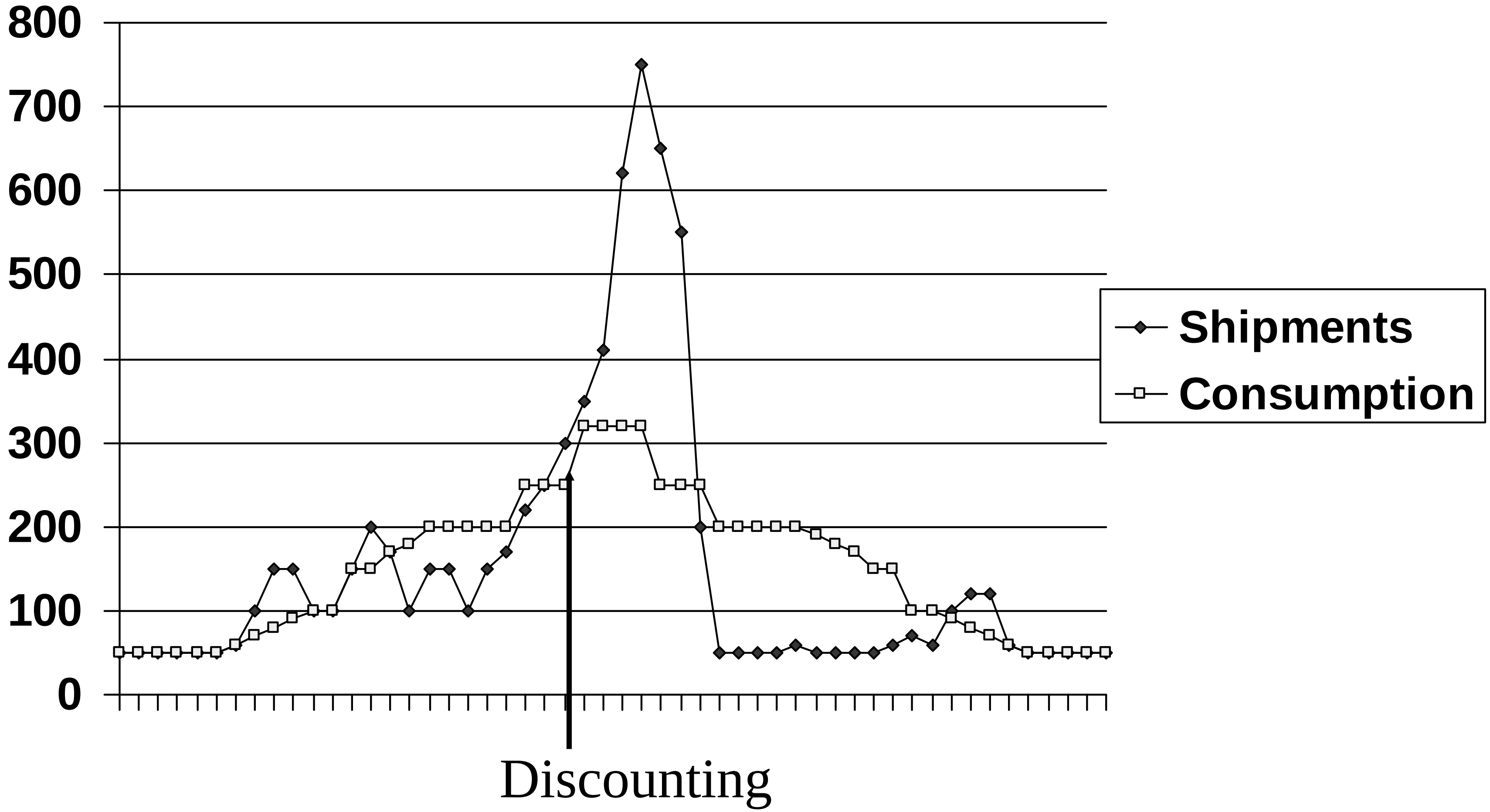
# Lessons From Discounting Schemes

- Lot size based discounts increase lot size and cycle inventory in the supply chain
- Lot size based discounts are justified to achieve coordination for commodity products
- Volume based discounts are more effective in general especially in keeping cycle inventory low
  - End of the horizon panic to get the discount: Hockey stick phenomenon
  - Volume based discounts are better over rolling horizon

# Short Term Discounting

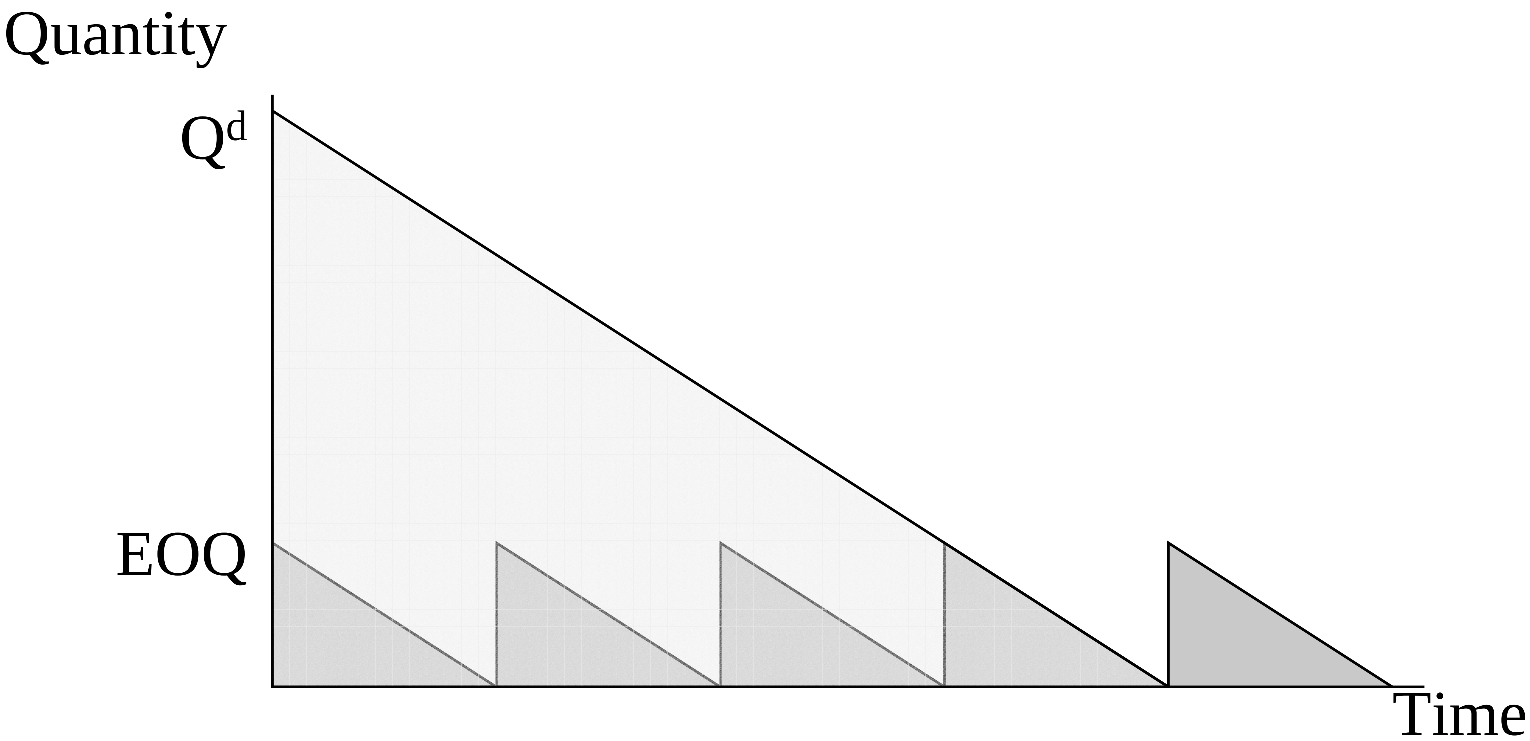
- Why?
  - To increase sales, Ford
  - To push inventory down the SC, Campbell
  - To compete, Pepsi
- Leads to a high lot size and cycle inventory because of strong forward buying

# Weekly Shipments of Chicken Noodle Soup. Forward Buying



# Short Term Promotions

- Promotion happens only once,
- Optimal promotion order quantity $Q^d$ is a multiple of EOQ



# Short Term Discounting

***C*: Normal unit cost**

***d*: Short term discount**

***R*: Annual demand**

***h*: Cost of holding $1 per year**

**$Q^d$: Short term (once) order quantity**

$$Q^{d} = \frac{d\ R}{(C - d)h} + \frac{C\ EOQ}{C - d}$$

**Forward buy = $Q^d$ - $Q^*$**

# Short Term Discounts: Forward buying. Ex 10.8 on p.280

Normal order size, EOQ = 6,324 bottles

Normal cost, $C$ = \$3 per bottle

Discount per tube, $d$ = \$0.15

Annual demand, $R$ = 120,000

Holding cost, $h$ = 0.2

$Q^d = 38{,}236$

Forward buy =38,236-6,324=31,912

Forward buy is five times the EOQ, this is a lot of inventory!

# Supplier's Promotion passed through to consumers

Demand curve at retailer: 300,000 - 60,000*p*

- Normal supplier price, $C_R$ = \$3.00

Retailer profit=(p-3)(300,000-60,000p)

  - Optimal retail price = \$4.00
  - Customer demand = 60,000

- Supplier's promotion discount = \$0.15, $C_R$ = \$2.85

Retailer profit=(p-2.85)(300,000-60,000p)

  - Optimal retail price = \$3.925
  - Customer demand = 64,500

- ***Retailer only passes through half the promotion discount and demand increases by only 7.5%***

# Avoiding Problems with Promotions

- Goal is to discourage retailer from forward buying in the supply chain
- Counter measures
  - Sell-through: Scan based promotions
    - Retailer gets the discount for the items sold during the promotion
  - Customer coupons; Discounts available when the retailer returns the coupons to the supplier. The coupons are handed out to consumers by the supplier. Retailer realizes the discounts only after the consumer's purchase.

# Strategic Levers to Reduce Lot Sizes Without Hurting Costs

- Cycle Inventory Reduction
  - Reduce transfer and production lot sizes
    - » Aggregate the fixed costs across multiple products, supply points, or delivery points
      - E.g. Tailored aggregation
  - Are quantity discounts consistent with manufacturing and logistics operations?
    - » Volume discounts on rolling horizon
    - » Two-part tariff
  - Are trade promotions essential?
    - » Base on sell-thru (to consumer) rather than sell-in (to retailer)

# Inventory Cost Estimation

- Holding cost
  - Cost of capital
  - Spoilage cost, semiconductor product lose 2% of their value every week they stay in the inventory
  - Occupancy cost
- Ordering cost
  - Buyer time
  - Transportation cost
  - Receiving/handling cost
- Handling is generally Ordering cost rather than Holding cost

# Summary

- EOQ costs and quantity
- Tailored aggregation to reduce fixed costs
- Price discounting to coordinate the supply chain
- Short term promotions